Modified Cohesive Crack Model Adapted for Description of Interface Fracture at Micro, Meso and Macro-Scales

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Summary

One of the basic assumptions underlying all cohesive crack models used in the description of inelastic fracture has to do with the shape of the cohesive force distribution. The exact form of this distribution is unknown, but several very useful clues are provided by the experimental work on fracture at interfaces, cf. Hutchinson [1]. In principle it could be derived from considerations of the molecular forces exchanged between two adjacent planes of atoms which are subject to separation as the leading edge of the crack propagates along the interface.

We shall return to this point after some mathematical preliminaries. The condition of finite stress at the tip of the extended crack, $|x|<a$ (a visible crack stretches along $|x|<c$), valid for the stress boundary conditions

$$p(x) = \begin{cases} \rho, o < X < C \\ \rho - S(x), c < X < a \end{cases}$$

(1)

can be set up as follows

$$0 = K_{TOT}(\sigma,S) = 2\sqrt{\frac{a}{\pi}} \int_0^a \frac{p(x)dx}{\sqrt{a^2 - x^2}} =$$

$$= 2\sqrt{\frac{a}{\pi}} \left\{ \int_0^c \frac{\sigma dx}{\sqrt{a^2-x^2}} + \int_c^a \frac{[\sigma - S(x)]dx}{\sqrt{a^2-x^2}} \right\} =$$

(2)

$$= 2\sqrt{\frac{a}{\pi}} \left\{ \pi\frac{\pi}{2} + \int_c^a \frac{[-S(x)]dx}{\sqrt{a^2-x^2}} \right\}$$

If the stress distribution $S(x)$ is normalized by the reference cohesive stress S_0, say $S(x) = S_0 G(x)$, then Eq. (2) reduces to

$$Q = \int_c^a \frac{[-G(x)]dx}{\sqrt{a^2 - x^2}}, Q = \frac{\pi\sigma}{2S_0} \qquad (3)$$

When the variable x is replaced by x_1, $x = x_1 + c$, Eq. (3) reads

$$Q = \int_0^R \frac{G(X_1)d X_1}{\sqrt{a^2 - (X_1 + C)^2}} \qquad (4)$$

or, better yet

$$Q = \int_0^1 \frac{G(\lambda)(1 - m)d\lambda}{\sqrt{1 - [(1 - m)\lambda + m]^2}} \qquad (5)$$

Here, $\lambda = x_1/a$ and m is a parameter related to the crack length c and the length of the extended crack, a = c + R, namely, m = c/a. In what follows we shall limit the considerations to the case of R<<c, i.e., for m→ 1, which is pertinent for "small scale yield condition" met in all cases of practical importance in the context of Materials Science. For this limiting case the integral in Eq. (5) can be simplified as follows

$$[Q_{(m)}]_{m \to 1} = \int_0^1 \frac{1 - m}{\sqrt{1 - m^2}} \frac{G(\lambda)d\lambda}{\sqrt{1 - \lambda}} = \sqrt{\frac{1 - m}{2}} \int_0^1 \frac{G(\lambda)d\lambda}{\sqrt{1 - \lambda}} \qquad (6)$$

Valuable clues regarding the distribution $G(\lambda)$ are gained from studies of fracture occurring at the interface between two dissimilar materials joined together either by direct adhesion or by a thin bonding film. In order to account for the experimental data, two main features are expected. First, the stress S should reach a maximum at a certain distance Δ from the crack front. This maximum stress S_{max} may in some cases become substantially larger than the reference stress S_0. It is assumed that S_{max} is attained somewhere within the process zone, most likely at its outer edge, $x_1 = \Delta$. To the left of this point S drops off rapidly to zero to match the boundary condition of stress-free crack at $x_1 = 0$, while to the right of this point S falls down again and levels out at the value S_0, toward the end of the cohesive zone, $x_1 = R$.

In order to account for such behavior we propose a strongly nonlinear function composed of a power function and an exponential. We submit, therefore, a two-parameter distribution function of this form

$$S(X_1) = S_0 X_1^n \exp\left[\alpha\left(1 - \frac{X_1}{R}\right)\right], X_1 = R\lambda \tag{7}$$

$$G(\lambda) = \lambda^n \exp[\alpha(1-\lambda)], 0 \le \lambda \le 1$$

in where α and n are yet undetermined parameters. This function is now substituted into Eq. (6), yielding

$$Q_m = \sqrt{\frac{R}{2C}} \int_0^1 \frac{\lambda^n \exp[\alpha(1-\lambda)]}{\sqrt{1-\lambda}} d\lambda \tag{8}$$

Note that for $m \to 1$, the expression $(m-1)$ can be replaced by R/c, while the integral in Eq. (8) can be cast into a closed form, cf. [2]

$$W(\alpha, n) = \frac{1}{\Gamma\left(\frac{3}{2}+n\right)} \left[\sqrt{\pi} \exp(\alpha)\Gamma(n+1) = F_1\left(1+n, \frac{3}{2}, -\alpha\right)\right] \tag{9}$$

Here the standard notation for the gamma function (Γ) and the hypergeometric function (F_1) is used, cf. [3]. Physical interpretation of the integral (9) leads to the energy dissipated within the cohesive zone, hence the symbol W. Finally, combining Eqs. (8) and (9) allows us to define the length of the cohesive zone

$$R = \frac{\pi}{2W^2}\left(\frac{K_I}{S_0}\right)^2 \tag{10}$$

When K_I attains its critical level K_{Ic}, the Eq. (10) predicts the characteristic microstructural length parameter, $R_{max} = (\pi/2W^2)(K_{Ic}/S_0)^2$.

The primary conclusions of this contribution can be summarized as follows:

(1) A generalization has been proposed that encompasses all previous cohesive crack models and provides a platform for novel investigations of the influence of the structured nature of the nonlinear zone on the early stages of fracture.

(2) By proper choice of parameters α and n we are able to quantify the inner structure of the cohesive zone, the so-called "fine structure", which accounts for the existence of the small process zone of size Δ embedded within the larger R-zone.

(3) Microstructure of material is now represented by properties such as the overstress factor, $k = S_{max}/S_0$ and the ductility parameter, $\rho = R_{ini}/\Delta$, in which R_{ini} denotes the threshold value of R associated with the onset of fracture.

For a given k and ρ, the parameters that determine the shape of the S-distribution, α and n, can be evaluated explicitly by matching the ratio $S_{max}/S_0 = (n/\alpha)^n \exp(\alpha - n)$ with the given overstress factor, k. Solving the equation

$$\left(\frac{n}{\alpha}\right)^2 \exp[\alpha - n] = k \qquad (11)$$

for the coefficient α, we obtain

$$\alpha = \frac{\rho}{\rho - 1} \ln\left(k \rho^n\right) \qquad (12)$$

Since α/n represents the reciprocal of the coordinate λ at which the maximum in S occurs, we have

$$\frac{\alpha}{n} = \frac{1}{\lambda_{max}} = \frac{R_{ini}}{\Delta} = \rho \qquad (13)$$

Combining it with Eq. (12) results in transcendental equation

$$\frac{\rho}{n(\rho - 1)} \ln\left(k \rho^n\right) - \rho = 0 \qquad (14)$$

For any given input set of data, such as specified ρ and k, the other two variables, α and n, can be solved for (numerically, of course). Since the input parameters are deduced from the microstructural data, and can be measured experimentally, the fine structure characteristics α and n are not accessible to an experiment, we have provided a link between the two sets of parameters pertaining to micro-level of fracture. The next step, of course, is to evaluate the macro–level entities such as W and R. Our model makes these calculations possible, too. And thus, we have indeed constructed a bridge between the micro- and macro-scales of fracture representation.

To illustrate this statement, we set $\rho = 10$ and $k = 5$, and then using the equations written above, we obtain $n = 0.2403$ and $\alpha = \rho n = 2.4031$, while the nondimensional dissipation of energy for those microstructural input data is $W (\alpha, n) = 4.4805$, and the length of the nonlinear zone is $R = 0.3506(K_{Ic}/S_0)^2$.

Finally, Figure 1 shows the predicted shape of the G-function, which represents a nondimensional cohesive force distribution within the R-zone for the choice of micro-parameters used in our sample calculation.

References

1. J. W. Hutchinson, 1997, "The Role of Plasticity in Toughening of Ductile Metals and Interfaces", Seminar at Northwestern University in the series "Colloquia on Modern Topics in Mechanics", March 1997, Evanston, IL.
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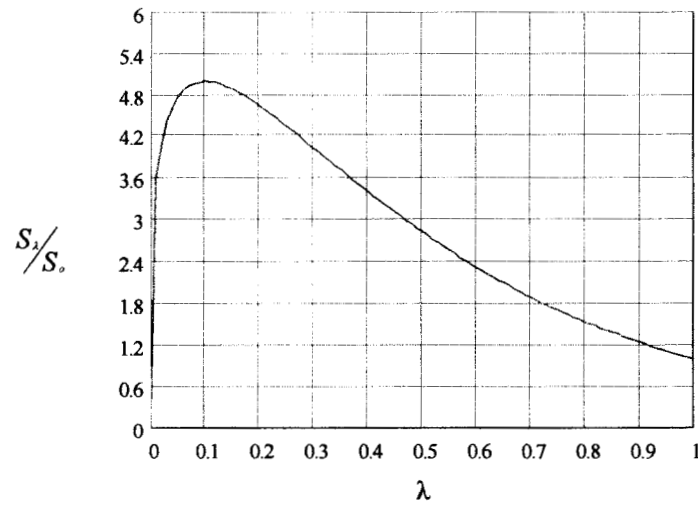
Figure 1: Distribution of the cohesive force $S(\lambda)/S_0$ within the R-zone for the following meso-structural parameters:- ductility index, $\rho = 10$, and - overstress factor, $k = 5$.